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METHOD FOR DETERMINING FATIGUE CHARACTERISTICS OF METALS UNDER COMBINED BENDING AND TORSION

PETER R. PERKINS

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Peter R. Perkins

METHOD FOR DETERMINING FATIGUE CHARACTERISTICS
OF METALS
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by

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Submitted in partial fulfillment
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fatigue of metals under repeated combined bending and torsional stresses. His excellent work was summed up in his Presidential Address to the Institution of Mechanical Engineers given in 1949. (Reference A). He has designed and constructed two elaborate machines for testing one half inch metal specimens under combined bending and torsional loadings. His machine is capable of producing sinusoidal loading (pure bending or pure torsion, or any combination of both) over a wide range of frequencies.

The work of Dr. Gough has been little expanded upon by other researchers. It was my desire to devise a simple method of obtaining reversed bending and torsional moments over a range of frequencies using the Westinghouse Vibration Fatigue Equipment available at the U. S. Naval Postgraduate School, and to apply these moments to metal specimens in fatigue.

The purpose of this paper is to present a method of obtaining combined alternating stresses using the Westinghouse equipment. Although this method was not tested in its entirety (only the reversed bending portion was constructed and tested) it is believed that small specimens of restricted design can be tested successfully by a procedure similar to the one herein presented.

The guidance, assistance, and encouragement of Professor E. K. Gatcombe has been invaluable and is acknowledged with many thanks. The skillful machine work which contributed materially to the success of the experiments was due to Mr. R. P. Kennicot. An acknowledgement is made to all the members of the faculty of the Postgraduate School who have directly or indirectly contributed to the author's background and development.

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TABLE OF SYMBOLS AND ABBREVIATIONS

x_1	Absolute amplitude of motion of main system
x_2	Absolute amplitude of motion of center of specimen
δ	Center displacement of specimen with respect to its supports (sometimes 2δ)
x_{st}	Static deflection of system with driving force applied
P_o	Driving force
Ω_n	Natural frequency of main mass system (motor spider, etc.) radians per second
ω	Driving frequency in radians per second
K	Spring constant of main mass system (spider, etc.)
k	Spring constant of specimen
d	Diameter
l	One half the length of specimen between bearings
$d_{\mathbb{C}}$	Diameter of specimen at center line where it is necked when for maximum stress
d_{av}	Average diameter of specimen assumed for certain simplifications
M	Moment
σ	Bending stress
τ	Shear stress
θ	Angle of twist of half the specimen
f_{n_b}	Specimen natural frequency in bending (cycles per second)
f_{n_t}	Specimen natural frequency in torsion (cycles per second)
ω_{n_b}	Specimen natural frequency in bending (radians per second)
ω_{n_t}	Specimen natural frequency in torsion (radians per second)
W_b	Bending weight
W_t	Torsion weight
r_d	Radial distance between torsion weights and center of specimen

μ Mass ratio, absorber mass/main mass

f Frequency ratio

g Forced frequency ratio

c Viscus damping coefficient of specimen

c Radial bearing clearance

c_c Critical damping of specimen

p Bearing oil pressure in pounds per square inch.

e Eccentricity of bearing journal

n Attitude

S_o Dynamic Summerfield Number

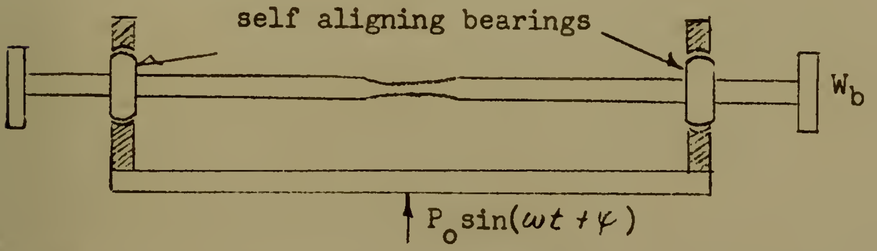
I Moment of Inertia of cross section, a necked down section, or center line.

CHAPTER I

INTRODUCTION AND CONCLUSIONS

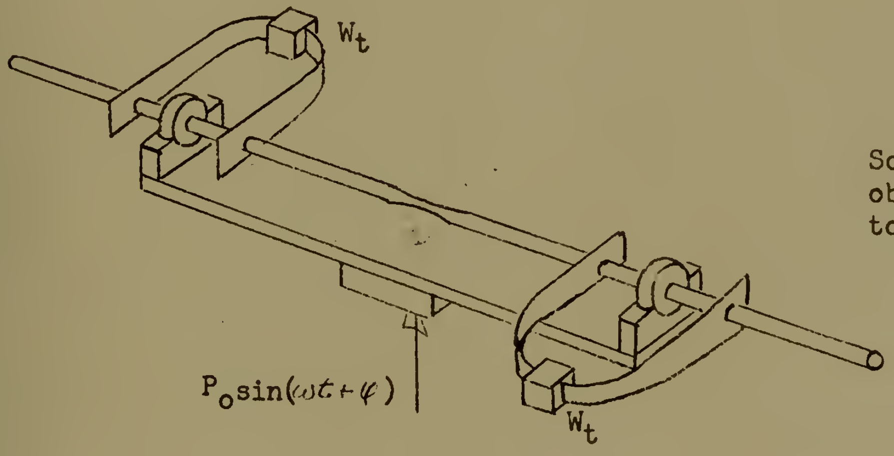
1. Outline of Proposed Method.

Is there a simple scheme whereby combined cyclic bending and torsion can be placed upon a metal test specimen? If a specimen could be oscillated through a known distance in space, weights on either end of the specimen could provide inertia forces. Pure bending moments can be obtained if the specimen is held as follows: (see also figure 1)



Scheme for obtaining pure bending.

Pure torsional moments can be obtained by placing weights on opposite sides of the specimen as shown in the following sketch.



Scheme for obtaining pure torsion.

The above two arrangements can be combined giving simultaneous bending and torsion if the supports are free to revolve in all directions, devoid of lost motion, and strong enough to transmit the necessary forces.

Spherical bearings were designed to provide this necessary freedom of motion in all directions. The journal would not rotate to any noticeable extent, so special attention must be given to design it for minimum drag and adequate life. The bearings were forced lubricated (from oil ports in both the top and the bottom of each bearing) with light oil at 120 pounds per square inch pressure. The oil flowed out the sides of the bearings where it was collected and piped back to the supply tank. In operation the specimen was completely oil supported at all times and the only drag encountered came from the viscous properties of the oil. Clearances were kept small and no lost motion or eccentric movement was noticeable to the touch or from the pickup signal. These spherical bearings were clamped to opposite ends of a cradle which was caused to vibrate by the Westinghouse Vibration Motor.

2. Frequency and Size Considerations.

An exhaustive exploration of fatigue characteristics of any metal would require that effects at all frequencies be examined. However, little variation is encountered below 83 cycles per second. (Variations from 5 to 7 per cent are observed at the higher frequencies. "Influence of Testing Frequency on Fatigue Strength" by T. Wyss [11]) Therefore it would be desirable to test specimens at frequencies below this frequency so as to be able to compare the test results with previously obtained and confirmed data. There would be a decided advantage at operating in this area due to the fact that small variations in the frequencies of successive runs will be permissable. Thus the experimenter can purposely adjust the driving frequency to the resonant frequency of the system, and obtain maximum stresses. Operations at these lower frequencies limits the diameter of the speci-

men to less than one half an inch. These slender fatigue specimen are unfortunately subject to size effects and results evaluated accordingly. Thus a balance between the two (frequency effects versus size effects) must be sought, or one of the effects accepted outright. I chose to accept the size effect and designed the specimen with a diameter of .25".

3. Conclusion

Calculations show the distinct possibility of obtaining cyclic bending and torsional stresses from the above mentioned set up powered by the Westinghouse Vibration Fatigue Equipment. Experimental reversed bending runs, using a jig manufactured at the U. S. Naval Postgraduate School gave excellent sinusoidal bending of the specimen. It is believed that torsional stressing could also be readily obtained and excellent data on combined bending and torsional fatigue be plotted as described in Chapter II.

It is pointed out that static bending and torsional loads could be superimposed upon the sinusoidal fluctuations, giving vent to a much wider range of investigation.

CHAPTER II

GENERAL EXPERIMENTAL PROCEDURE

Any specimen will have two controllable natural frequencies - the bending and the torsional frequencies. The specimen to be tested should be so designed that the stress from either the bending moments or torsional moments will be sufficient to break the specimen. Once a specimen design is decided upon it will be possible to control the torsional or bending stresses by (a) varying the natural frequencies of the specimen by changing the bending or torsional masses (as resonant frequency is approached stresses will increase) or by (b) varying the magnitude of the vertical oscillations. An examination of the two methods of control indicates method (b) is the more useful.

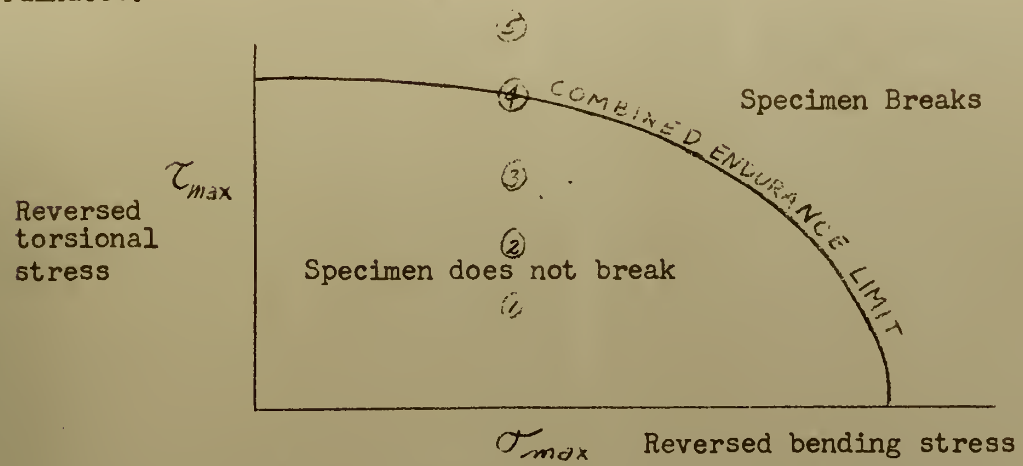
1. Varying the natural frequency of the specimen (ω_n) by reduction of weights to obtain desired stresses.

As previously mentioned the frequency of loading for any group of tests should be approximately the same frequency, even though this is not too important under 83 cycles per second. It would be kept constant while the natural frequency of the specimen were varied. By varying the natural frequency of the specimen the resonant point is approached, and the loading thus varied.

This would be a poor method of control due to the fact stress variation near the resonant frequency will be very critical. It would undoubtedly be very difficult to maintain the stress level constant during the run due to slight frequency variations that undoubtedly would be present.

The advantage of this approach is the ease of plotting data. A series of separate specimen with identical bending stresses can be subjected to

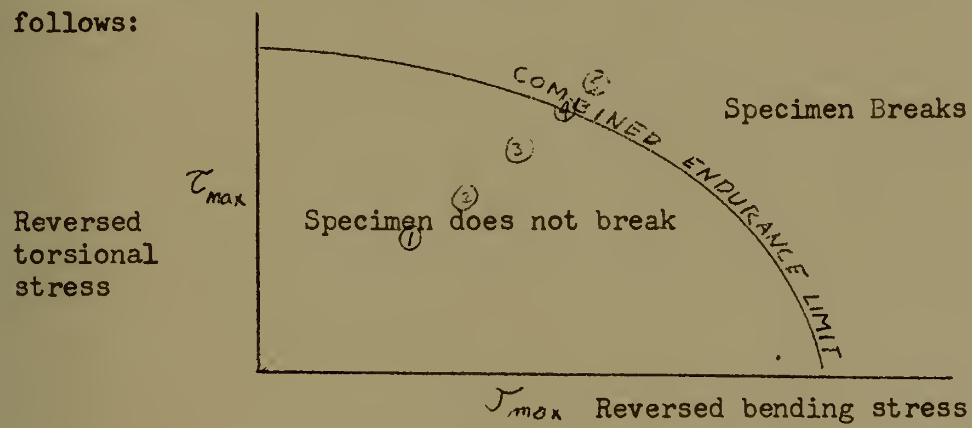
different (successively larger) torsional loads until the combined endurance limit is crossed. They would appear as follows on shear bending coordinates.



Endurance limits under combined bending and torsional sinusoidal loads.

2. Control by varying the magnitude of vertical oscillations.

Nothing would be varied during successive runs in this case except the magnitude of the driving force (or x_{st}). Therefore the torsional and bending stresses will increase together. Successive runs will give points as follows:



Endurance limits under combined bending and torsional sinusoidal loads.

The advantage of this is that the weights can be adjusted so that both the resonant torsional and bending frequencies are the same. Thus larger speci-

mens could be tested and departure from the critical size region is approached. Also operation over a flat portion of the resonant frequency will permit more stable operation. The damping present in the system will prevent the resonant frequencies from becoming narrowly delineated.

CHAPTER III

DESIGN

1. Purpose

In order to obtain maximum cyclic loads on a specimen using a limited source of power operation at the natural frequency of the system is decidedly advantageous. The following analysis is presented to show the procedure resorted to for determining whether a certain specimen can be broken with the equipment at hand.

2. Preliminary Design

If the specimen were a round rod .3" in diameter with a reduced neck in the center with a diameter of .25", a maximum stress of 28,600 pounds per square inch at the extreme fibers of the .3" section would give a maximum bending stress of 50,000 psi on the surface of the necked down section. This was adequate for failure of the material being tested, SAE 1040 steel.

From the relation $\sigma = \frac{MC}{I}$; $28{,}600 = \frac{M \quad .15}{3.9 \quad 10^{-4}}$ (1)

the moment M is readily found to be about 75 inch pounds.

If the specimen is about 6" long, and the short necked down section at the center is ignored, the following formula gives the deflection at the center.

$$\delta = \frac{M\ell^2}{2EI} = \frac{75 \cdot 3^2}{2 \cdot 30 \cdot 10^6 \cdot (3.9) \cdot 10^{-4}} = .0289" \qquad (2)$$

δ is the relative displacement of the center of the specimen with respect to the portion supported in the self aligning bearings at either end. The specimen can be likened to a spring (assuming small displacements). It would then be considered an absorber and would absorb much of the energy of the main vibrating system of the vibration motor. Absolute motion of the center of the specimen is x_2, while the motion of the supporting cradle is x_1.

The natural frequency of the vibration motor can be modified by the amount of mass attached to the vibrating shaft used to transmit the vibratory motion. A cross bar securely fastened to the stator of the vibration motor was used to steady the entire assembly. See figure 3.

Examination of the instruction book of the Westinghouse Vibration Motor showed that a forcing force of 75 to 300 pounds is available. A value of 120 pounds was selected as a conservative value to work with. If the spring constant of the cross bar is designed to be about 10,200 pounds force per inch, then the X_{st} of the cross bar will be .0117.

$$K = \frac{P_o}{X_{st}} \qquad \begin{aligned} P_o &= 120 \text{ pounds} \\ K &= 10,220 \end{aligned}$$

If the desired δ, or $x_2 - x_1 = .0289"$, and $x_{st} = .0117$, then the ratio $\frac{x_2 - x_1}{x_{st}} = \frac{.0289}{.0117} \approx 2.5$

The question now is can a magnification of x_{st} by a factor of 2.5 be obtained in the actual mechanical system? DenHartog, page 123, gives a formula for the displacement of the main mass, x_1 with respect to x_{st}. The expression for the relative displacement of the two masses (including the damping of main system) is

$$\frac{x_2 - x_1}{x_{st}} = \sqrt{\frac{g^4}{\left\{g\left(2\frac{c}{c_c}f^2 + 2\frac{c}{c_c}f\right) - 2g^4\left[(c + C)\mu + c\right]\right\}^2 + \left\{g^2\left[\mu f^2 + \frac{4f}{c_c}(2c + C)\right] + (g^2 - 1)(g^2 - f^2)\right\}}}$$

Taking representative values for u, $\frac{c}{c_c}$, and f a curve was plotted showing the relative displacement of the two masses with respect to g, $\left(\frac{\omega}{\Omega_n}\right)$. Such work indicated that a magnification of 2.6 was to be expected, (if of course the assumed values were of the correct order of magnitude). See Appendix I for the above work.

Before proceeding further an examination of the possibility of obtaining, in addition to the bending, a torsional stress adequate to fatigue a specimen at the running frequency will be made.

Looking at the previous equation it is noticed that the natural frequency ratio of the absorber to the motor, f, occurs in several terms of the denominator and has the following effect on the relative displacements $(x_2 - x_1)$. As f increases the relative displacement $\frac{x_2 - x_1}{x_{st}}$ becomes smaller. Therefore by varying the natural frequency of the specimen either in bending or torsion the resonant relative amplitude can be modified. Best results would be obtained if both the bending and torsional natural frequencies were identical so that the increase in relative displacements with the variation of the natural frequency of the motor will be available.

The following assumptions were made in the above design approach.

1. Neglected weight of specimen
2. Specimen is uniform rod .3" in diameter, 6" long.
3. Forcing force of 120 pounds is obtainable from motor.
4. Crossbar spring constant can be made to be about 10220.
5. See Appendix I for assumptions in making up curve of

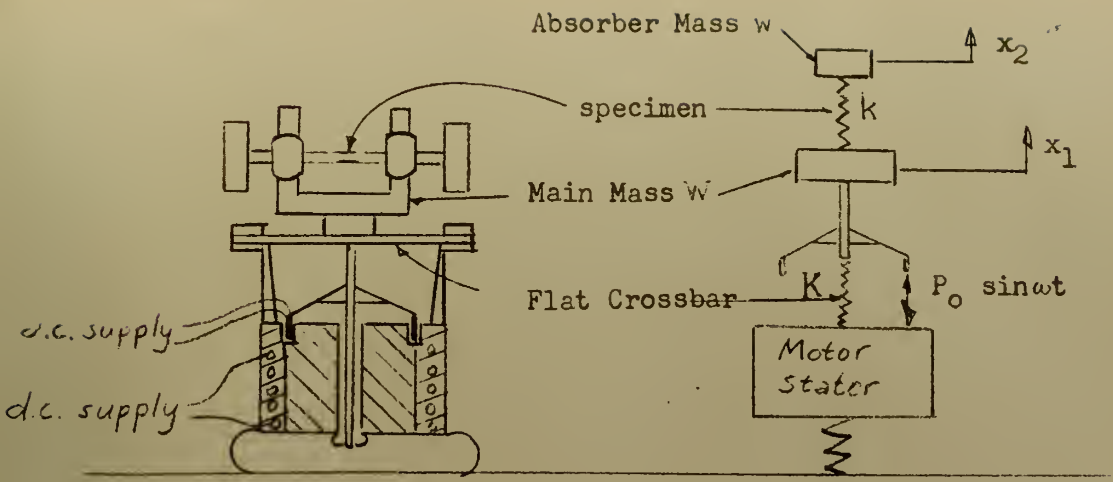
   $$\frac{x_2 - x_1}{x_{st}} \quad \text{versus } g.$$
6. That the vibration motor is still in inertial space.

3. Design of Elements

The following sketch indicates the assembly of the many elements.

Schematic arrangement of

vibration equipment.



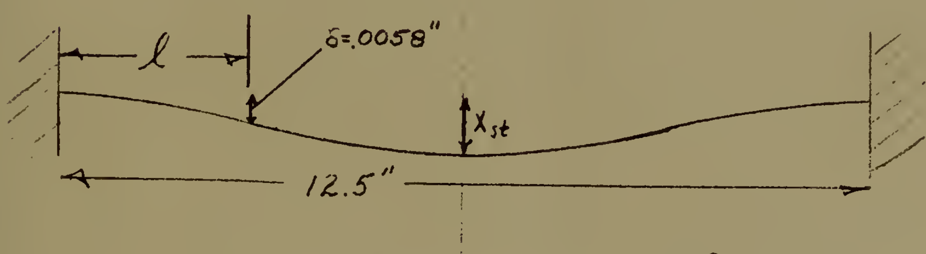
A. Design of Flat Cross Bar. Analysis in paragraph 2 specified that the cross bar should have a K = 10220.

If P_o = 120 lbs

$$K = \frac{P_o}{x_{st}}$$

$$10220 = \frac{120}{x_{st}}$$

$$x_{st} = .0117$$



For a cantilever beam $\delta = \frac{P\ell^3}{3EI}$

$$.0058 = \frac{60\ (3.12)^3}{3\ E \cdot \frac{bh^3}{12}}$$

(b) Width of beam has 3 inches h = .241 inches

The cross bar was made one quarter of an inch thick. Measurement of K after installation gave a value of 10220. Examination was made to see if the cross bar had adequate strength. Predicted stress was 6680 pounds per square inch, cyclic loading. This was considered to be a reasonable level of stress.

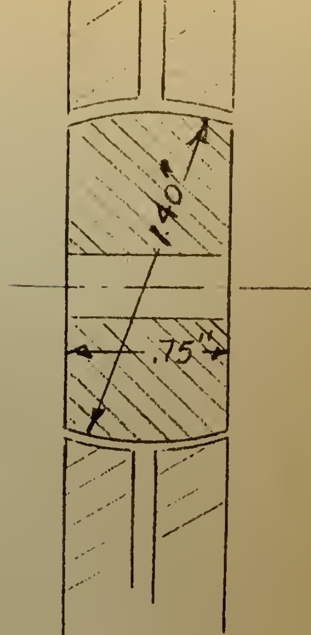
B. Design of Bearings. If the specimen has an overhanging weight at either end, it will be subjected to a bending moment. The desired moment is 75 inch pounds. If the weights overhang one inch then the vertical force necessary in the bearing to sustain such a moment is 75 pounds. The bending moment due to the "pivoting" inertia of the weights was assumed to be small.

The bearing was to be a solid spherical one for reasons previously mentioned. The shaft or journal was not designed to rotate. Conventional bearings would be unable to support a load under this restriction. Therefore oil was admitted to both the top and the bottom portions of the bearing under adequate pressure to assure its entry. The oil film would be able to sustain this oscillating load if properly designed. Good design should be able to limit the eccentric motion to less than one hundredth of the forcing motion.

The bearing journal was a machined ball, solid steel, seven tenths of an inch radius. The bearing was a matching sphere lapped to fit as closely as possible. The bending load of 75 pounds plus a torsional load of 112 pounds would give a maximum load of 187 pounds. The bearing was designed for an alternating load of 300 pounds as follows.

The bearing dimensions assumed were as indicated on the adjacent figure. Projected bearing area was taken as one square inch. Equation (6-52), page 218, ANALYSIS AND LUBRICATION OF BEARINGS, by Saw and Macks gave the maximum pressure that will be encountered in a bearing subjected to repeated loading. The equation is:

$$p = \frac{P\,(1-n^2)^{3/2}}{\pi n}\left[\frac{1}{(1+n\cos\theta)^2} - \frac{1}{(1+n)^2}\right]$$



Spherical
Support
Bearing

Then assuming the attitude (n) (ratio of eccentricity to radial clearance) is .2 the pressure is calculated as follows:

$$p_{max} = \frac{\frac{300}{(1\ in)^2}\,(1-.04)^{3/2}}{\pi\ .2}\left[\frac{1}{(1-.2)^2} - \frac{1}{(1+.2)^2}\right] = 404.1 \text{ psi}$$

The Dynamic Summerfield Number as given by Shaw and Macks is:

$$S_o = \left(\frac{r}{c}\right)^2 \frac{\mu\omega}{P_o}$$

Then assuming the radial clearance is .002" it is evaluated:

$$S_o = \left(\frac{.7}{.002}\right)^2 \frac{2\cdot 10^{-6}\ \ 440}{330} = .36$$

Entering the curves on page 220 of Shaw and Macks with this Summerfield Number the maximum attitude is picked off as .14

$$\text{attitude } n = \frac{\text{eccentricity}}{\text{clearance}} = .14 = \frac{e}{.002}$$

$$e = .00028 \text{ inches}$$

The percentage of "slap" created by the eccentric motion of the specimen in the bearing is of utmost importance and can now be approximated as $\frac{.00028}{.0117} = 2.39\%$. This estimate is a generous one and even so it would probably be unable to contribute sufficient disrupting inertia to be discernable.

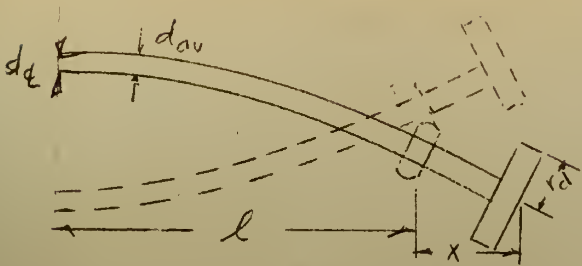
C. Design of Jig and Cradle. See figure 3. The bearing caps were secured by one high strength bolt, 3/8 inch, on each side. The bearings were drilled for oil passage, oil entering both the top and the bottom of the bearing under 120 pounds per square inch pressure, and leaving through the ends of the bearings. The oil then ran down into a drip pan, from which it returned to the sump. Oil was delivered to the bearing blocks through high pressure flexible hose.

The cradle supporting the two bearings was made of aluminum. This permitted addition of extra weight to the main mass until its resonant frequency was lowered to 80 cycles per second. A steel cross support of more rigid design would be better.

D. Design of Specimen. Objective: To design specimen so that the magnitudes of the bending and torsional weights for each specimen configuration would be physically adaptable to the test equipment and easily mounted on the specimen. To have the specimen's natural frequencies in the vicinity of 83 cycles per second or below. The specimen should be about .3" in diameter and have a center deflection of .0289; this keeps the system within the analysis of Chapter II.

Four general formulas were developed so that the relationship between the physical dimensions of the specimen and the sought after natural frequencies and resultant stresses could be visualized. Many specimen configurations were explored and these equations clearly pointed out the direction in which to make necessary changes.

Natural frequency equation for bending.



Cantilever concept of specimen.

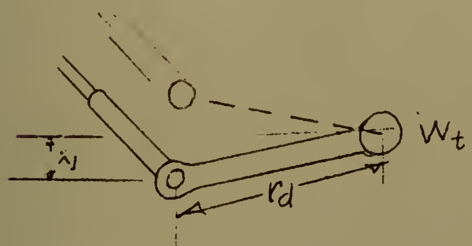
$$f_{n(b)} = \frac{7000\, d_{av}^{2}}{W^{\frac{1}{2}} \sqrt{r_d^{2} - 4x^{2}}} \qquad (3)$$

Center section will be horizontal due to symetry.

Bending stress equation is:

$$\sigma_b = \frac{60 \cdot 10^{6}\, C_{\ell}}{\ell^{2}} \left(\frac{d_{av}}{d_{\ell}}\right)^{4} \qquad (4)$$

The radius at which the torsion weight must be suspended:

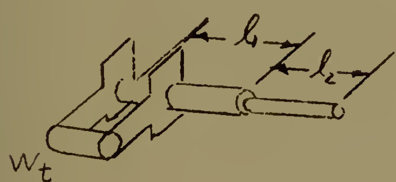


Torsional Weight on end of specimen.

$$r_d = \frac{X_1}{\tan \frac{\theta}{10}} \qquad (5)$$

$$\text{where } \theta = \frac{d_c^{3}}{3 \cdot 10^{6}} \left(\frac{\ell_1}{d_1^{4}} + \frac{\ell_2}{d_2^{4}}\right)$$

The natural torsional frequency $f_{n(t)}$:



Torsional weight on end of specimen.

$$f_{n(t)} = \frac{4120}{\left(\frac{\ell_1}{d_1^{4}} + \frac{\ell_2}{d_2^{4}}\right)^{\frac{1}{2}} W_t^{\frac{1}{4}}\, r_d} \qquad (6)$$

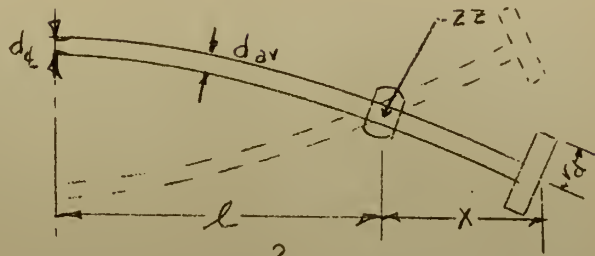
The preceding equations will now be developed.

Equation (3)

Development of an expression for the natural frequency of bending.

$$\theta = \frac{Ml}{EI}$$

$$k = \frac{M}{\theta} = \frac{EI}{\ell}$$



$$k = \frac{E}{\ell}\,\frac{d_{av}^{4}}{64} \qquad I_{weight} = \frac{m\,r_d^{2}}{4}$$

$$\omega_n = 2\pi f = \sqrt{\frac{k}{I_{zz}}} \qquad I_{zz} = \frac{m\,r_d^{4}}{4} - x^2 m$$

$$2\pi f = \sqrt{\frac{\dfrac{E\;R\;d_{av}^{4}}{1\;\;64}}{\dfrac{m\,r_d^{4}}{4} - x^2 m}}$$

$$f = \frac{7600\,d_{av}^{2}}{W^{\frac{1}{2}}\left[\ell(r^2 - 4x^2)\right]^{\frac{1}{2}}}$$

Equation (4)

Development of an expression for the stress expected.

$$\sigma = \frac{MC_{\text{℄}}}{I_{\text{℄}}} \qquad \delta_{rel} = \frac{M\ell^2}{2EI_{av}}$$

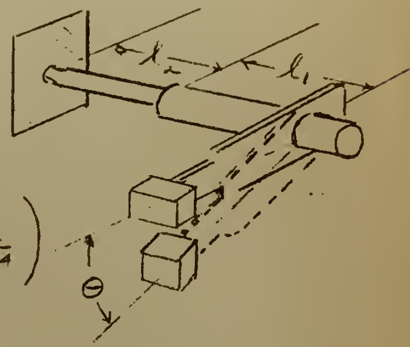
$$\sigma = \frac{2EI_{av}\;\delta_{rel}\;C_{\text{℄}}}{\ell^2\;I_{\text{℄}}}$$

$$= \frac{2\,E\;\cdot\delta_{rel}\,C_{\text{℄}}}{\ell^2}\left(\frac{d_{av}}{d_{\text{℄}}}\right)^4$$

Equation (5)

The desired shear stress is obtained by twisting the specimen in pure torsion through the angle θ. It has been previously reasoned how a resonant gain of 2.5 can be expected in bending. Similar reasoning indicates that a resonant increase of 2.5 could be expected in torsion. This assumption is dependent upon the damping that will be present in the torsional system.

This is where the low drag of the spherical bearings will be of most importance. The above assumption is also dependen~~t up~~on the mass ratio . A resonant increase of 2.5 is still a moderate assumption and little difficulty should be experienced, once the resonant frequency is adjusted to the resonant frequency of bending.



$$\theta = \frac{M_t l}{G I_p} \qquad \tau = \frac{16 M_t}{\pi d_t^3}$$

$$\theta = \frac{d_t^3 \tau l}{6 \cdot 10^6 d_{av}^4} \quad \text{or} \quad \frac{d_t^3 \tau}{6 \cdot 10^6}\left(\frac{l_1}{d_1^4} + \frac{l_2}{d_2^4}\right)$$

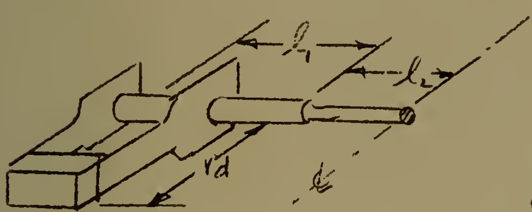
If the driving motion of the support is through the distance x_{st} (.0117") and the resulting angle of rotation of the specimen were $\frac{\theta}{2.5}$, then the resonant angle would be at least two and one half times this value, or

$$\tan\frac{\theta}{2.5} = \frac{.0117}{r_d}$$

$$r_d = \frac{.0117}{\tan\frac{\theta}{2.5}} \quad \text{where } \theta = \frac{d_t^3 \tau}{3.10^6}\left(\frac{l_1}{d_1^4} + \frac{l_2}{d_2^4}\right)$$

<u>Equation (6)</u>

Determination of relationship of specimen dimensions with the torsional natural frequency.



$$\omega^2 = (2\pi f)^2 = \frac{k}{I}$$

$$\theta = \frac{M_t l}{G I_p} \; ; \quad \frac{M_t}{\theta} = \frac{G I_p}{l} = \frac{G \pi d^4}{32 l} = k$$

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{32 l_1}{G \pi d_1^4} + \frac{32 l_2}{G \pi d_2^4}$$

$$k = \frac{1.177 \cdot 10^6}{\left[\frac{\ell_1}{d_1^4} + \frac{\ell_2}{d_2^4}\right]}$$

$$I_w = \left(\frac{w_t}{g}\right) r_d^{\,2}$$

$$(2\pi f)^2 = \frac{1.177 \cdot 10^6 \quad 386}{\left[\frac{\ell_1}{d_1^4} + \frac{\ell_2}{d_2^4}\right] w_t r_d^{\,2}}$$

$$f_{n_t} = \frac{4120}{\left[\frac{\ell_1}{d_1^4} + \frac{\ell_2}{d_2^4}\right]^{\frac{1}{2}} w_t^{\frac{1}{2}} r_d}$$

The weight necessary will be considerably less than above due to added torsional inertia effects of the bending weights.

CHAPTER IV

EXPERIMENTAL PROCEDURE

1. Operation of the Equipment.

The Westinghouse Vibration Motor must be in perfect adjustment prior commencement of runs. It consists essentially of a coil placed in a uniform magnetic field. The coil is rigidly fastened by means of a spider to a drive rod, which was in this case securely fastened to a flat cross bar one quarter of an inch thick. This flat cross bar was made to vibrate by the push rod. The jig to hold the specimen was securely bolted to the flat bar.

The instruction book leads one to believe that 500 watts can be put into the AC coil of the vibration motor, but due to its low impedance 172 watts was the maximum it could take without exceeding the 50 ampere limitation, under the arrangement shown in figure 3.

2. Recording and Measuring Equipment.

A Hewett Packard Audio Oscillator produced the controlling frequency. Excellent frequency stability was obtained throughout the run.

The automatic amplitude control of the Amplifier Exciter was not used during the runs. A continuous watch of the strain behavior throughout the run showed no change in the strain level. (Strain gages were attached to give specimen strains.)

Changes in the modulus of elasticity of the specimen during tests was not observed. If E does vary during the test due to fatigue behavior the stress will change accordingly. Thus this test procedure has the advantage of making realistic fatigue tests that would take into consideration the changes of modulus of elasticity of certain metals under fatigue loadings.

This method is unable to test a specimen independent of this effect.

The special shutdown control circuit built in the type Hi-40 Amplifier Exciter was not used. It is believed that it could be adapted to shut off the equipment when the specimen stress level drops off due to fracture of the specimen. It could get its signal either from the amplified strain gage signal, or perhaps from the pickup coil wound on the spider just for such a purpose. The amplitude of vibration of the main mass, or the spider, changes when the specimen breaks and is no longer capable of acting as an absorber. It was noted that the spherical bearings hold the specimen together after fracture, and that there is no visual evidence of fracture except the faint crack around the specimen.

The frequency of vibration was set on the audio oscillator and easily recorded. This equipment was calibrated prior to commencement of runs.

Strain gages were placed on the specimen and the amplitude of vibration varied until the desired stress level was reached. The strain gages were Baldwin Type A7. One was mounted on the top and the other mounted on the bottom of the specimen. They were connected to the terminals of the Ellis Associates Model BA-1 Bridge Amplifier. The output of the amplifier was placed on a Tektronix Inc Type 512 Cathode Ray Oscilloscope.

Strain gages were also placed on the flat cross beam. They were mounted one quarter of an inch from the clamped end supports.

Strain gages were also mounted on the vertical drive rod to obtain information on the magnitude of the forcing function.

The strong magnetic field set up by the stator of the vibration motor created induced voltages in the strain gages. Therefore each set of gages had to be mounted in such a manner so as to keep the induced voltages to a

minimum. The field in the vicinity of the specimen was weak due to the shielding effect of the flat cross bar. Also these gages were mounted parallel to the stronger of the flux lines. The gages mounted on the flat cross bar did not move appreciably since they were placed very close to the fixed support. The gages on the vartical drive rod were in a strong field but they were aligned parallel to the lines of flux. No check was made to determine the effect of the voltages induced in the strain gages.

It would have been interesting to observe the phase relationship between the specimen motion and the flat cross bar motion. Also the phase relation of the driving force with the specimen motion. From this data the energy distribution in the various systems could be nicely tied down and a very ready control over the test procedure might be the result.

CHAPTER V

EXPERIMENTAL RESULTS

1. Vibration Motor and Main Mass System.

The driving coil and spider were securely attached to the flat cross bar and all of the specimen holding equipment was fastened in place. A five and one half pound mass was bolted onto the cross bar. See figure 3. With everything assembled except the actual specimen and its weights the vibration motor was turned on. The frequency was varied until the resonant frequency was clearly observed. The resonant frequency of the vibration motor with all of the operating equipment in place was 80.5 cycles per second.

The K of the vibration motor was determined as follows. A 71.5 pound weight was placed on the beam deflecting it .007 inches. The K of the system is then $\frac{P}{\delta} = \frac{71.5}{.007} = 10220$

From the relationship $\Omega_n = \sqrt{\frac{K}{M}}$

$$M = .04 \text{ sluggs}$$

$$W = 15.45 \text{ lbs.}$$

The vibratory motion of the stator of the main system due to the fact the entire motor is mounted on springs was assumed to be minor and is not considered.

2. Specimen Constants.

In order to measure the natural frequency of the specimen mounted in the running position, it was placed in position on the vibration motor. The springs between the motor and the cross bar were locked together so that there would be no interference due to vibration of the main system. Baldwin

type A7 strain gages were mounted on the specimen and their signal placed upon an oscilloscope. The oil pressure in the spherical bearings was set at 120 pounds per square inch, the running value.

After all preparations were made the specimen was struck by a rubber hammer and the die away curve as presented on the oscilloscope was photographed. The damped natural frequency (q_n) was measured as 477 radians per second (76 cycles per second). The logarithmic decrement $l_n\left(\frac{x_n}{x_n - 1}\right)$ $= \delta =$ .1 This was measured from the die away curve. The damping constant of the specimen was computed as follows:

$$\delta = \frac{\pi C}{I q_d}$$

$$.1 = \frac{\pi C}{.01488 \cdot 477}$$

$$C = .225 \qquad \frac{c}{c_c} = \frac{\delta}{2\pi} = \frac{.1}{2\pi} = .0159$$

$$C_c = 14.5$$

The spring constant k was found:

$$q = \sqrt{\frac{k}{I} - \frac{c^2}{4I^2}}$$

$$477^2 = \frac{k}{.0149} - \frac{.225^2}{4(.0149)^2}$$

$$k = 3381$$

The undamped natural frequency was obtained next:

$$\omega_d = \sqrt{\frac{k}{I}} = \sqrt{\frac{3381}{.0149}} = 477 \text{ radians}/_{\text{sec}} = 76 \text{ cps}$$

3. Specimen Material Properties.

One half inch bar stock of SAE 1040 steel was used for specimen material Independent hardness tests showed it to have a brinell hardness of approximately 140. Two separate tension tests were run, both of which gave an ultimate of 82,300 pounds per square inch, and an E of $29.9 \cdot 10^6$. See figure 6.

Referring to Lipson, Noll, and Clock "Stress and Strength of Manufactured Parts" [6], the endurance limit for a machine member of this material, increased for size effect, was given as 37,000 pounds per square inch. Placing this estimate on log log paper, figure 7, indicates failure should occur around 90 to 100 thousand cycles if the reversed stress applied is set at 46,000 pounds per square inch.

Two separate specimens were vibrated at 70 cycles per second with a completely reversed loading resulting in a stress of 46,000 pounds per square inch at the outer fibers. One specimen broke at82,700 cycles and the other at 79,200 cycles of reversed bending. These results are relatively close together indicating the repeatability of test data, and comparing moderately well with the expected fatigue limit.

4. Running Data

The specimen was not run to failure in run number 3. It was left intact for future investigation by interested persons following.

Run No.	Time Start	Time Stop	Total Cycles	Amps	Operating Frequency	Units on Scope of Calibrating Step	BA-1 Multiplier
1	1637	1700	96,600	32	70	5.6	100
2	2116	2138	79,200	35	70	4.2	50
3	- -	- -	- -	Watts 52	70	5.7	50

Run No.	BA-1 Strain No. from graph	Strain per Step	Double Amplitude of Strain Signal	Quadrupled Strain	Strain E at .3" dia.
1	60 μ in/in	6000 μ in/in	3.3 units	3540	884 μ in/in
2	60 μ in/in	6000 μ in/in	4.9 units	3500	875 μ in/in
3	60 μ in/in	6000 μ in/in	6.5 units	3420	854 μ in/in

Run No.	Stress under gages	Stress at center line	Moment M	Deflection of flat bar	Rod Force	Specimen $x_1 - x_2$	x_{st}	$\frac{x_2-x_1}{x_{st}}$
1	26,500 psi	45,900 psi	69 in.lbs					
2	26,250 psi	45,600 psi	68.2in.lbs					
3	25,500 psi	44,700 psi	66.2in.lbs	007	141 lbs	.024	.0138	1.74

5. Comparison of Results with Data Computed.

	bending natural freq.	bending weights lbs.	torsion natural freq.	torsion weight arm length	torsion weight lbs.
Computed values	75cps	2	80cps	.86"	9.9 (max)
Experimental values	76cps	2	(No experimental torsional runs were made.)		

The resonant frequency of the system was very close to 79 cycles per second. See figure 8. This was close to the expected value of 80.5 cycles per second as determined from the theoretical curve of $\frac{x_2 - x_1}{x_{st}}$. However due to the large increase in the driving force P_o at frequencies slightly below 71 cycles per second ($g = .9$) (figure 9) the maximum specimen deflection occurred at 70 cycles per second (figure 10).

The unpredictable output of the vibration motor makes it difficult to design for maximum efficiency. It is recommended that an electrical analysis of the motor also be made and its effect included in the overall performance, vibration wise. If the experimenter has adequate knowledge of the electrical and physical parameters he will be able to use the equipment at hand much more effectively.

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1) $Mx_1 + Kx_1 + Cx_1 + k(x_1 - x_2) + c(x_1 - x_2) = P_0 \sin \omega t$

2) $mx_2 + k(x_2 - x_1) + c(x_2 - x_1) = 0$

1a) $-M\omega^2 x_1 + Kx_1 + k(x_1 - x_2) + j\omega \left[C(x_1 - x_2) + Cx_1\right] = P_0$

2a) $mx_L\omega^2 + k(x_2 - x_1) + j\omega \left[c(x_2 - x_1)\right] = 0$

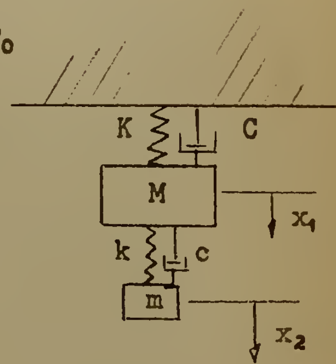
1b) $-M\omega^2 x_1 + Kx_1 + kx_1 + j\omega cx_1 + j\omega cx_1 - kx_2 - j\omega cx_2 = P_0$

1c) $\left[-M\omega^2 + K + k + j\omega c + j\omega c\right] x_1 - \left[k + j\omega c\right] x_2 = P_0$

2c) $-\left[k + j\omega c\right] x_1 + \left[-m\omega^2 + k + j\omega c\right] x_2 = 0$

1d) $Ax_1 - Bx_2 = P_0$

2d) $-Cx_1 + Dx_2 = 0$



$$\frac{x_2 - x_1}{P_0} = \frac{C - D}{(AD-BC)} = \frac{m\omega^2}{(AD-BC)}$$

where $(AD - BC) = \left\{(m\omega^2 - k)(M\omega^2 - K) + (km + 2c^2 + cC)\omega^2\right\} +$
$j\omega \left\{cK + Ck - \left[(c + C)m + cM\right]\omega^2\right.$

$$\left(\frac{x_2 - x_1}{P_0}\right)^2 = \frac{E^2 + F^2}{G^2 + H^2} \quad \text{where} \quad \frac{x_2 - x_1}{P_0} = \frac{E + jF}{G + jH} \text{ as above}$$

$$\left(\frac{x_2 - x_1}{P_0}\right)^2 = \frac{m^2\omega^4}{\left[(m\omega^2 - k)(M\omega^2 - K) + (km + 2c^2 + cC)\omega^2\right]^2 + \omega^2\left\{Ck + ck - \left[(c+C)m + cM\right]\omega^2\right\}}$$

Manipulation of the previous equation by taking numerator and denominator and 1) dividing by ω^4

2) dividing by M^2m^2

3) dividing by $\Omega_n^{\ 4}$

4) Multiplying by g^4

and using the following equalities

$$c_c = 2m\omega_n$$

$$C_c = 2M\Omega_n$$

$$X_{st} = \frac{P_o}{K}$$

results in this final equation

$$\left(\frac{x_1 - x_2}{X_{st}}\right)^2 = \frac{g^4}{\left\{g\left(2\frac{C}{C_c}f^2 + \frac{2cf}{c_c}\right) - \frac{2g^4}{c_c}\left[(c+C)\mu + c\right]\right\}^2 + \left\{g^2\left[\mu f^2 + \frac{4f}{C_c}(2c+C)\right] + (g^2-1)(g^2-f^2)\right\}^2}$$

The following values were used in plitting curves

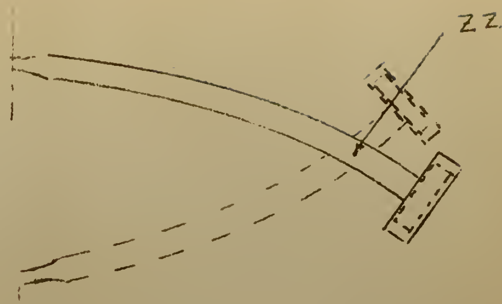
$C = .645$ $\quad$ $c = .157$ $\quad$ $f = \frac{\omega_n}{\Omega_n} = .945$

$C_c = 40.5$ $\quad$ $c_c = 10$ $\quad$ $\Omega_n = 80.5$

$\frac{C}{C_c} = .016$ $\quad$ $\frac{c}{c_c} = .016$ $\quad$ $\omega_n = 76$

$$\mu = \frac{w}{W} = \frac{4.5}{15} = .33$$

Calculation of Specimen Natural Frequency



$$2\pi f = \sqrt{\frac{k}{I}}$$

$$I_{wt} = I_{ring} = \frac{.945}{386/12}\left[3(2.4)^2 + 3(2.2)^2 + .8^2\right] = .0066$$

$$I_{disc} = \frac{1}{386/4}(2.2)^2 = .00313$$

$$I_{wt} = .0097$$

$$I_{wt_{zz}} = .0097 + r^2 m = .0097 \cdot 1^2 \cdot \frac{2}{386} = .01488$$

$$2\pi f = \sqrt{\frac{3350}{.01488}} = 474 \text{ radians}$$

$$f = 75 \text{ cycles per minute}$$

Note: Formula No. 4 was for solid weights. The above procedure was used to calculate the natural frequency due to the fact the weights used were hollow.

1. Calculation to determine length of arm to hold the torsion weights so adequate shear strain is obtained.

$$\Theta = \frac{d_{\mathrm{c}}^{3}\,\tau\left(\frac{\ell_1}{d_1^4}\right)}{6\cdot 10^6}$$

$$\Theta = \frac{\frac{1}{64}\ 35{,}000\ \ 3}{6\cdot 10^6\ \ 21\cdot 10^{-4}} = .0338 = 1.93^\circ$$

$$r_d = \frac{.0117}{\tan \frac{1.93}{2.5}} = \frac{.0117}{.0136} = .86''$$

2. If the natural torsional frequency of 80 cycles per second is desired, the following calculation is performed to determine the size of the torsional weight necessary.

$$f_{n_t} = \frac{4120}{\left(\frac{\ell_1}{d_1^4} + \frac{\ell_2}{d_2^4}\right)^{1/2} W_t^{1/2}\, r_d}$$

$$80 = \frac{4120}{\left[\frac{3}{21\ 10^{-2}}\right]^{1/2} W_t^{1/2}\ .86''}$$

$$W_t = 9.8\ \text{lbs}$$

This value appears to be excessive and specimen design changes may be necessary to reduce it to an acceptable value.

Appendix IV

Deflections and Loadings

Drive Freq.	"g" ω/Ω_n	Motor Input Watts	Specimen $\epsilon_{3''}$ μin/in	Specimen $\sigma_{.3}$ p.s.i.	Specimen $\sigma_{.25}$ p.s.i	Specimen M in lbs	Specimen X_2-X_1 inches	Specimen $\frac{X_2-X_1}{X_{st}}$	Flat Cross Bar $\epsilon_{.25}$ μin/in	Flat Cross Bar $\sigma_{.25}$ p.s.i	Flat Cross Bar X_1 inches	Drive Rod ϵ μin/in	Drive Rod σ p.s.i	Drive Rod P_o lbs.	Drive Rod X_{st} inches
65	.808	56	657.5	19,650	34,200		.018	1.078	278	8300	.0145	88	2630	171	.0167
70	.87	52	854	25,500	44,700	662	.0238	1.73	145	4320	.0070	72.5	2170	141	.0138
75	.93	60	828	24,700	43,000		.023	3.77	50	1490	.0026	33.5	970	63	.0061
80	.995	64	828	24,700	43,000		.023	4.49	108	3120	.0055	27	807	52.5	.00513
85	1.055	58	697	20,800	36,700		.0194	1.27	157	4680	.0081	80.5	2400	156	.0153
90	1.12	56	612	18,200	31,700		.017	1.45	162	4830	.0085	61.8	1850	120	.0117
95	1.18	60	354	10,608	18,500		.010	1.46	125	3740	.0065	36.2	1080	70	.00685

Table of deflections of specimen and flat cross bar, and driving force of drive rod.

Phase relationships were not obtainable due to instrumentation limitations.

Fig. 1. Fatigue specimen with bending weights (the large discs) pressed on each end. The nut holding them on is to maintain them in the correct axial position. The two balls are pressed on the shaft and are the journals which ride in the support bearings.

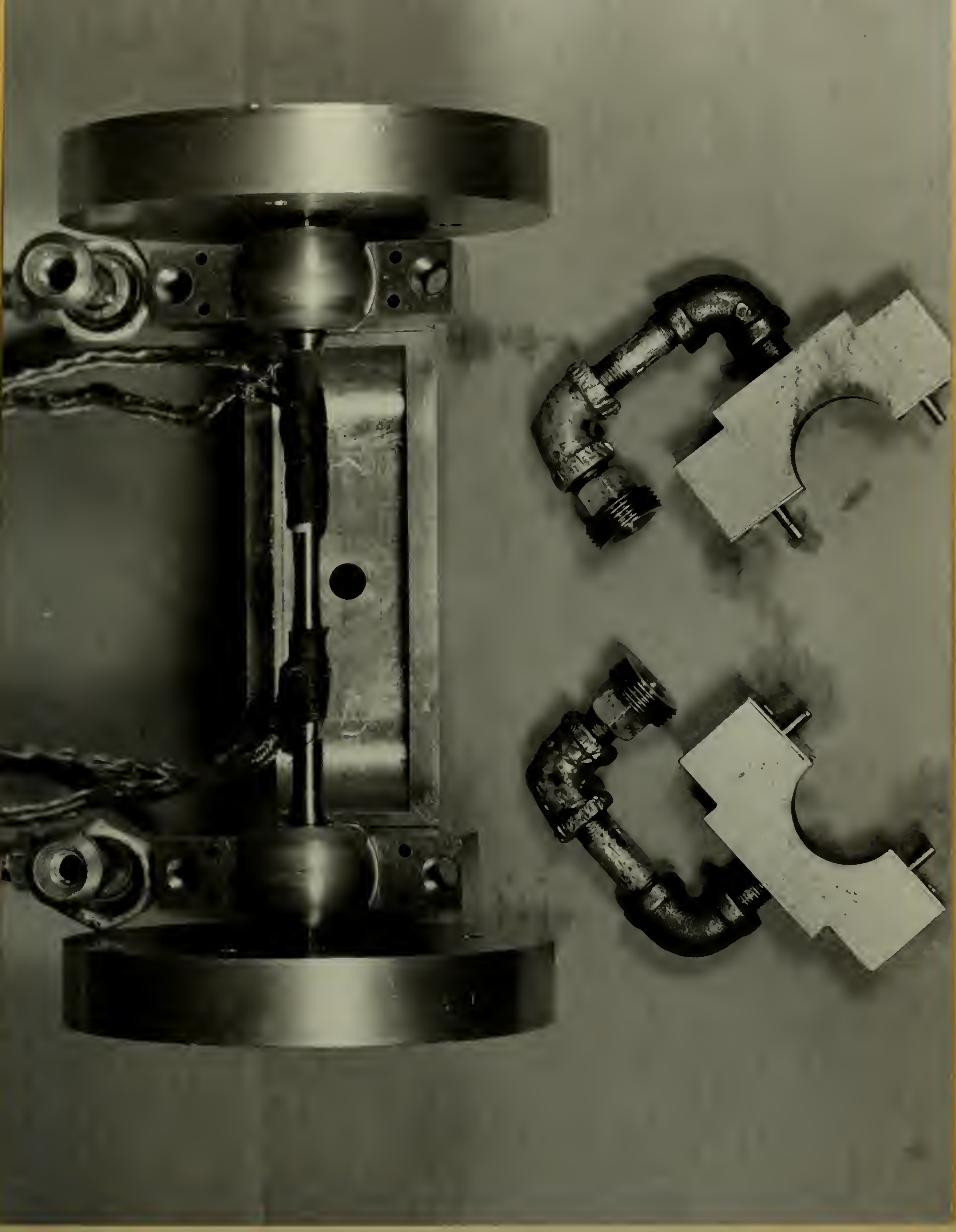


Fig. 2. Fatigue specimen in jig. The bearing caps are removed and are in the foreground. Note the oil cupply lines that go to both the top and bottom half of the bearings. Strain gages have been attached and are covered with tape.

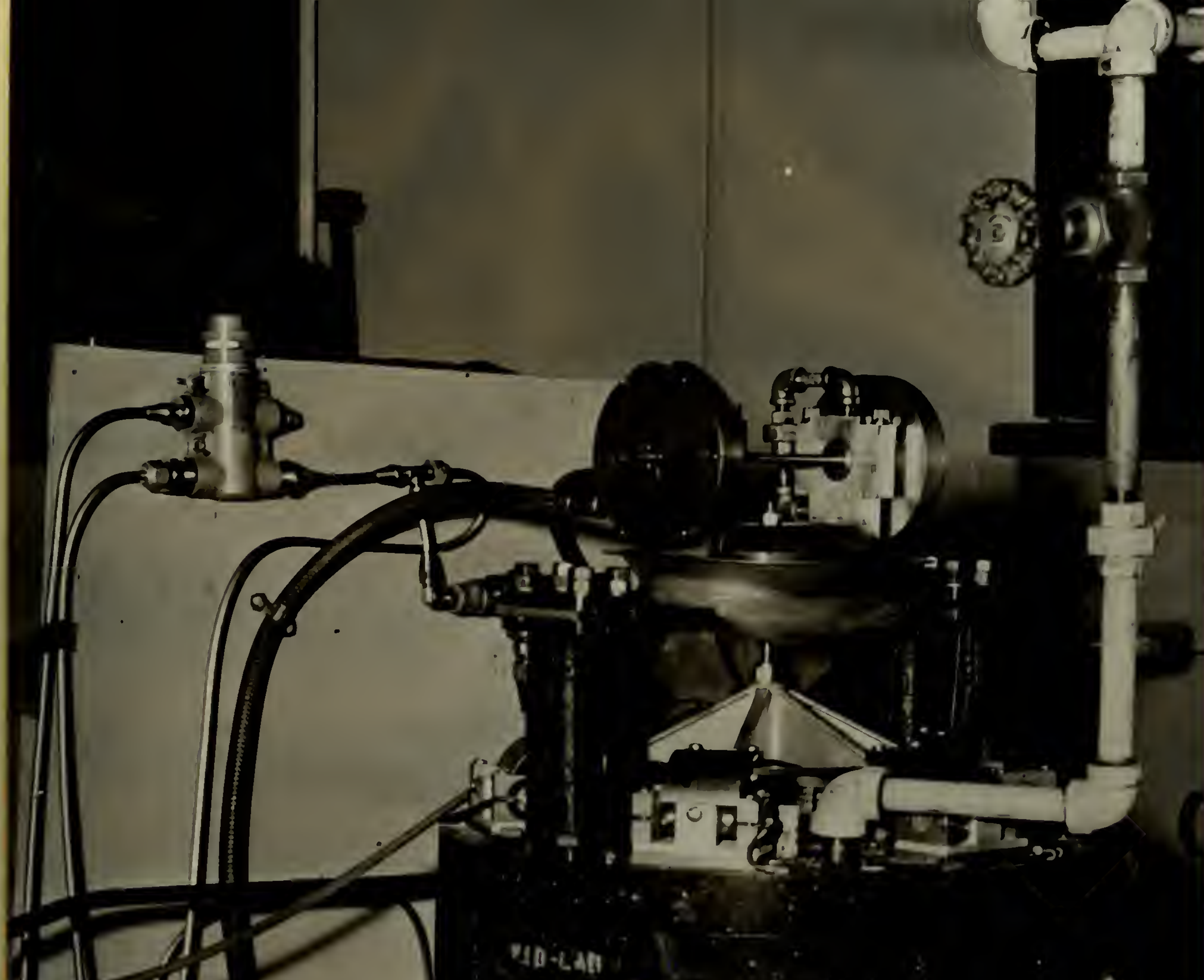


Fig. 3. The vibration motor with flat cross bar, oil pan, jig and specimen all in place. The flexible oil hose can be seen coming in from the rear.

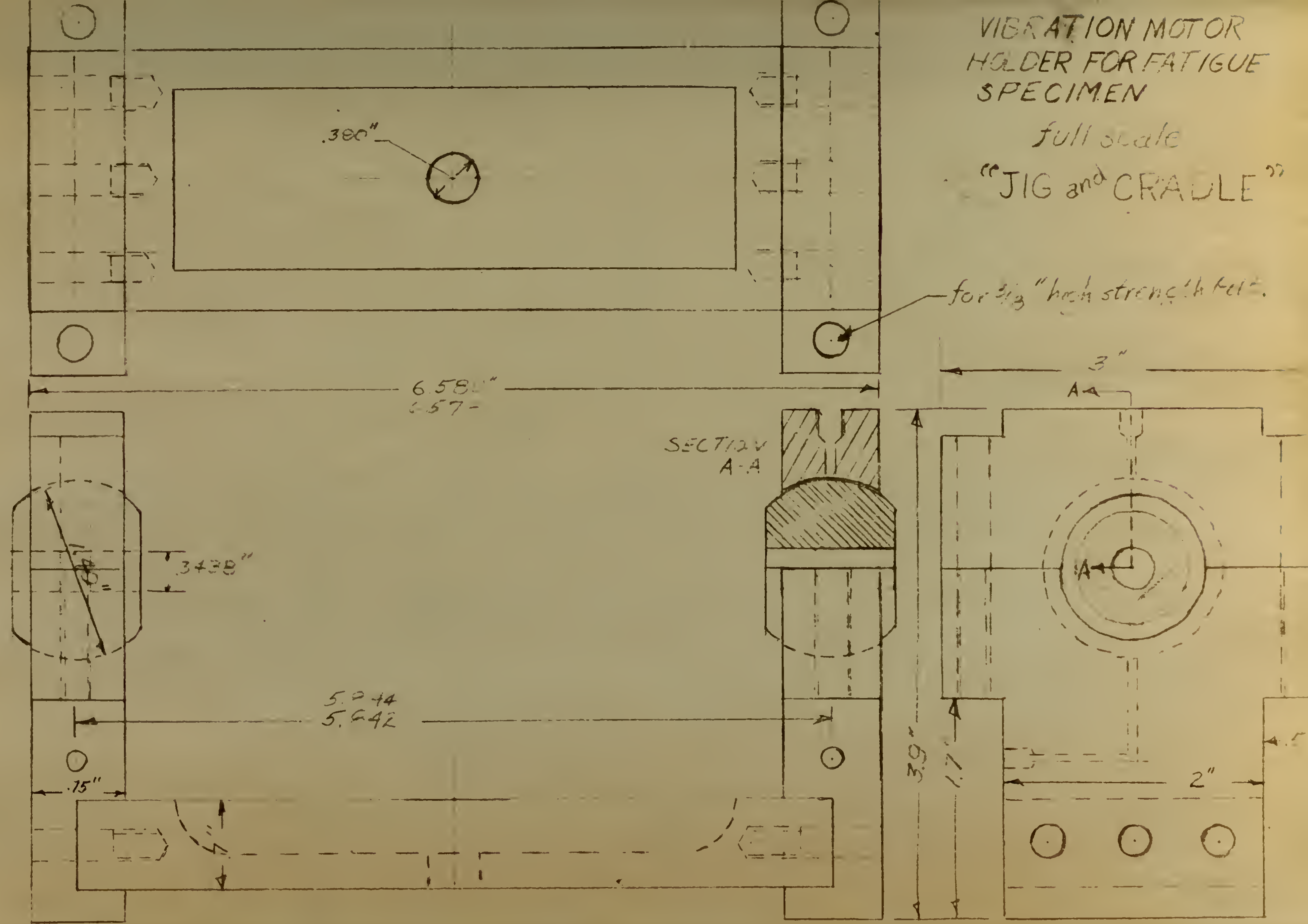

VIBRATION MOTOR
HOLDER FOR FATIGUE
SPECIMEN
full scale
"JIG and CRADLE"
for 1/3" high strength bolt
.380"
6.58"
6.57
SECTION
A-A
3"
A
.3438"
5.944
5.942
.15"
2"
3.9"
1.7"

FIG 4

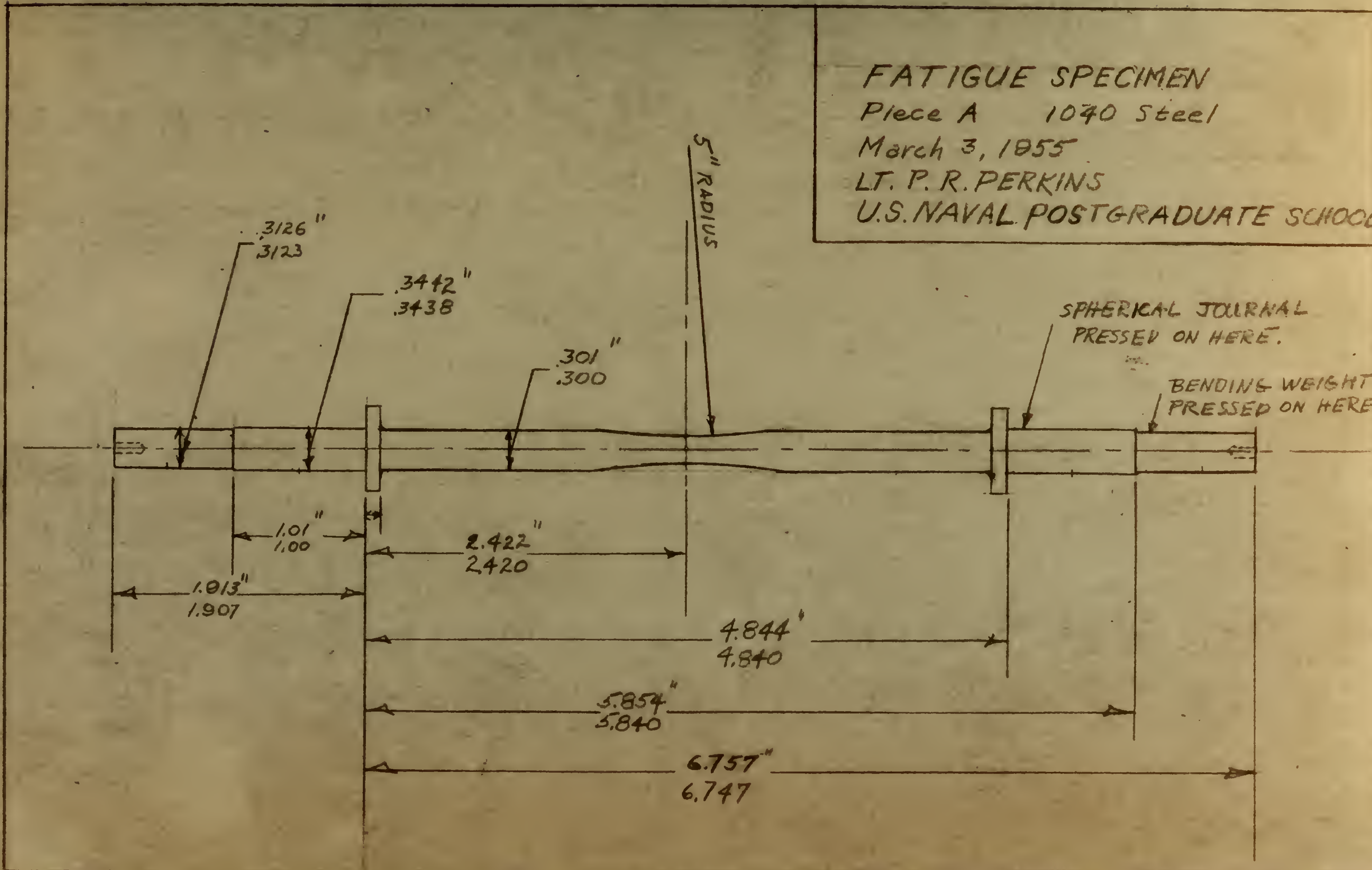
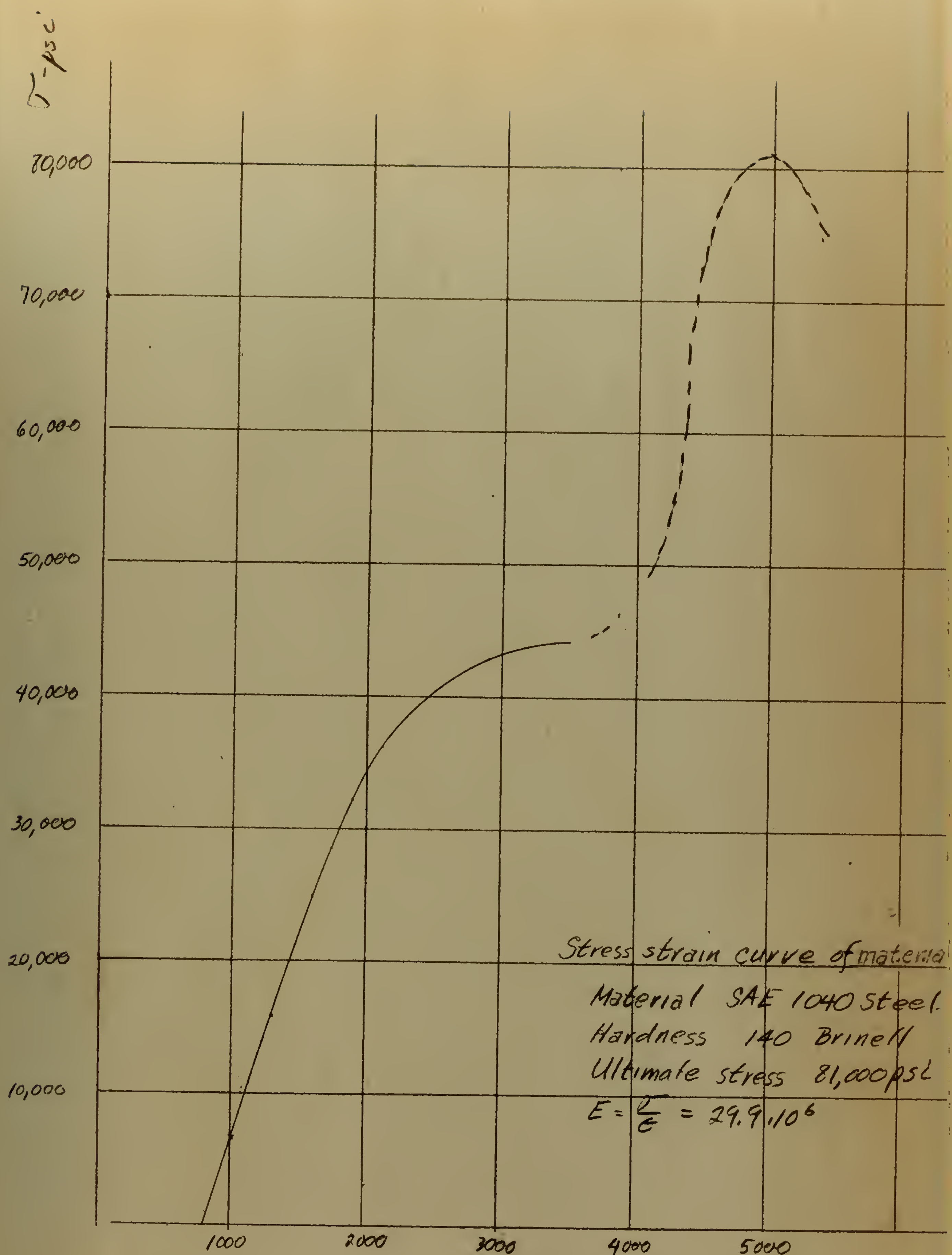

FATIGUE SPECIMEN
Piece A 1040 Steel
March 3, 1855
LT. P. R. PERKINS
U.S. NAVAL POSTGRADUATE SCHOOL
.3126" .3123
.3442" .3438
.301" .300
5" RADIUS
SPHERICAL JOURNAL PRESSED ON HERE.
BENDING WEIGHT PRESSED ON HERE.
1.01" 1.00
2.422" 2.420
1.913" 1.907
4.844" 4.840
5.854" 5.840
6.757" 6.747

FIG 5



σ - psi
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
1000
2000
3000
4000
5000
ε micro inches/inch
Stress strain curve of material
Material SAE 1040 Steel.
Hardness 140 Brinell
Ultimate stress 81,000 psi
E = σ/ε = 29.9·10^6

Fig 6

Fatigue Strength for Finite Life

Made from data found in Lipson, Knoll, & Clock for material similar to specimen tested.

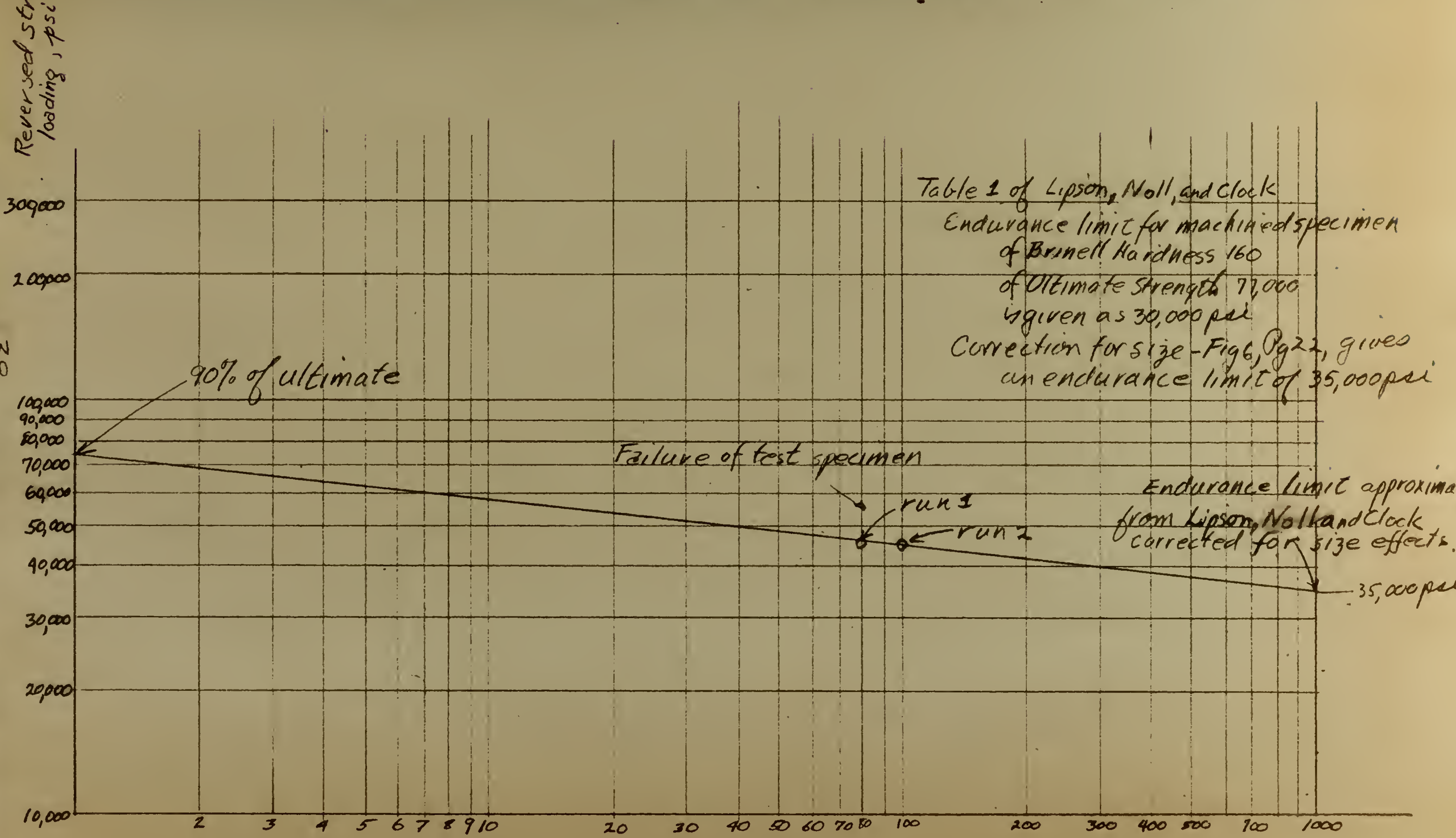
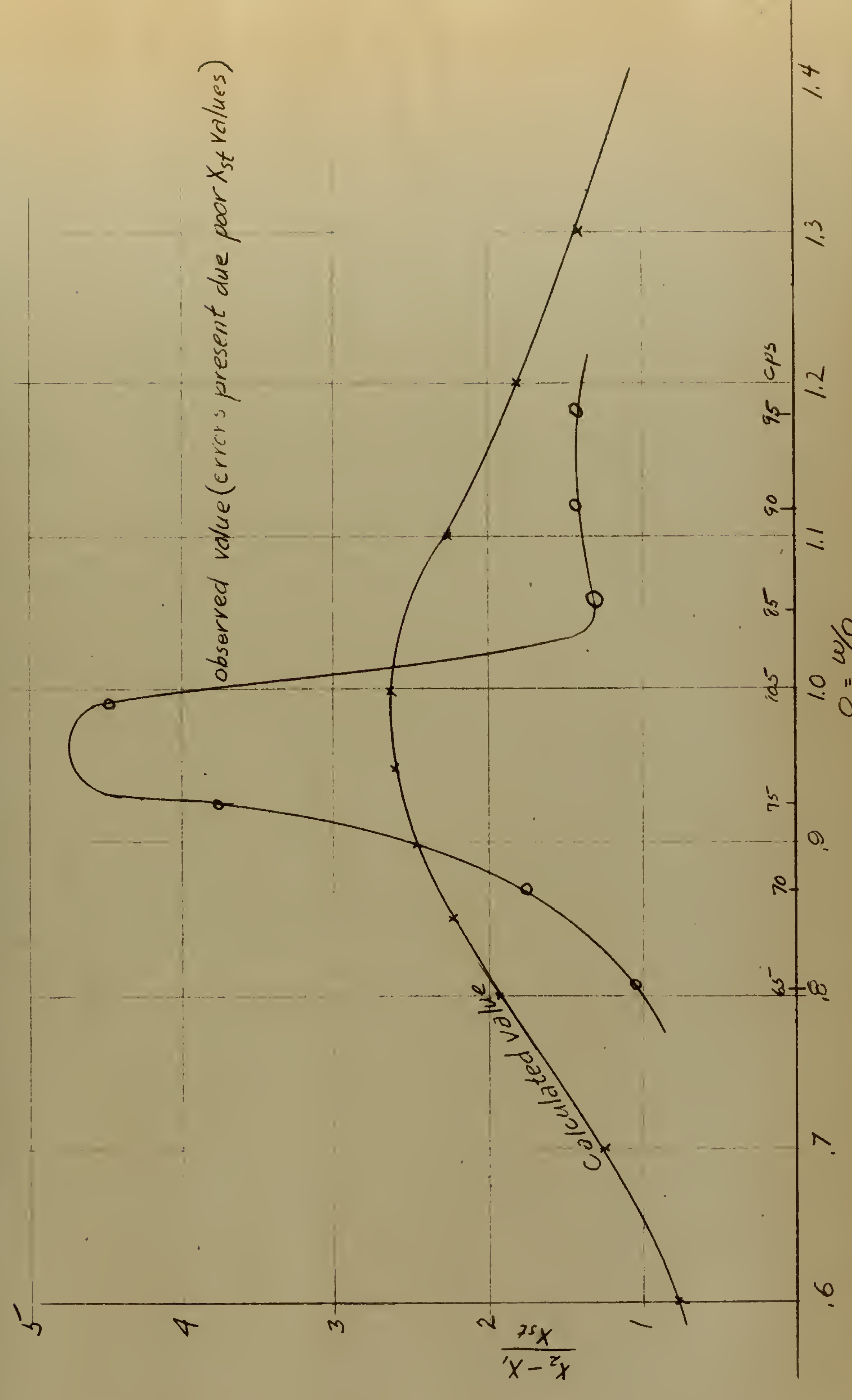
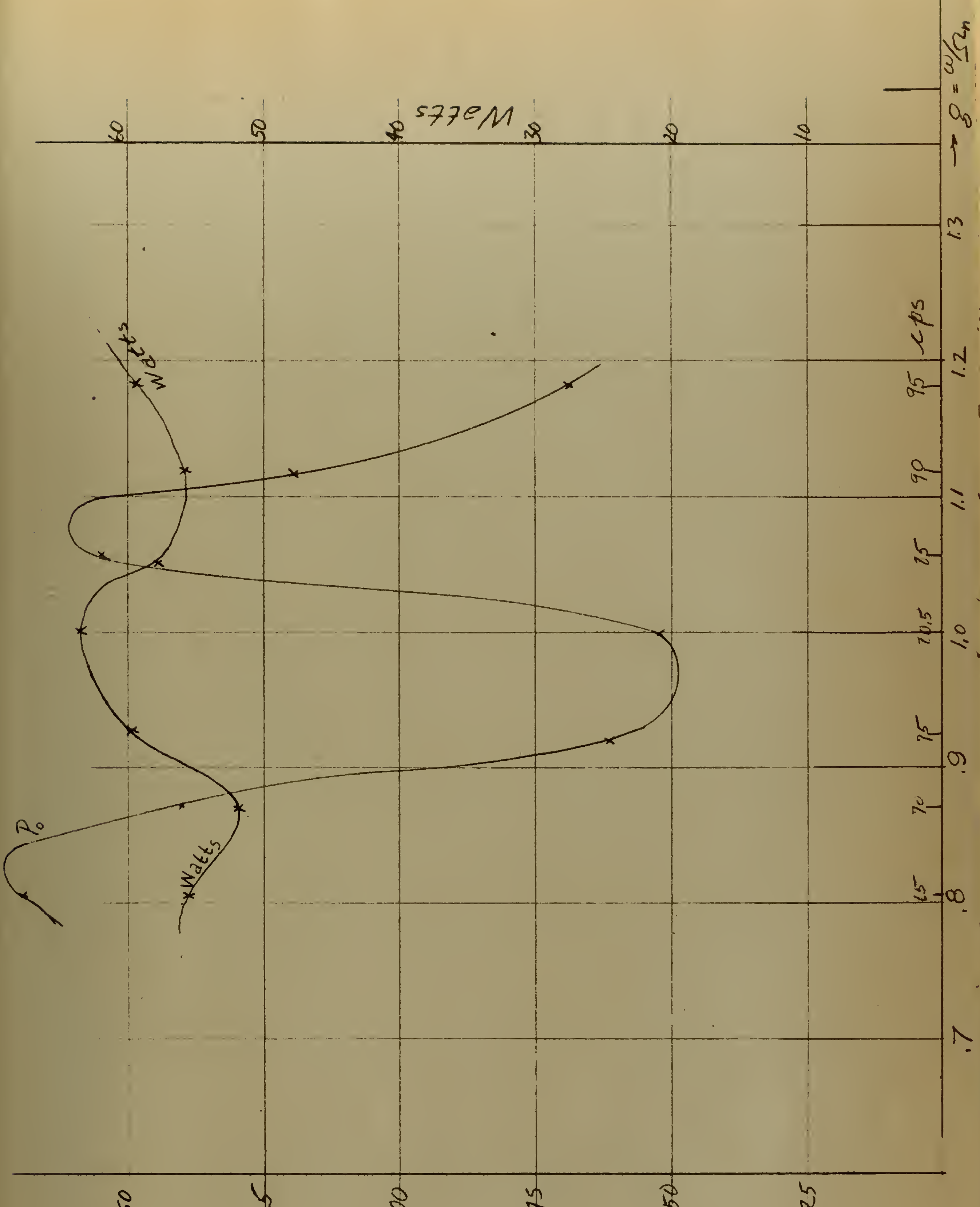


Fig 7 1000 cycles of reversed bending



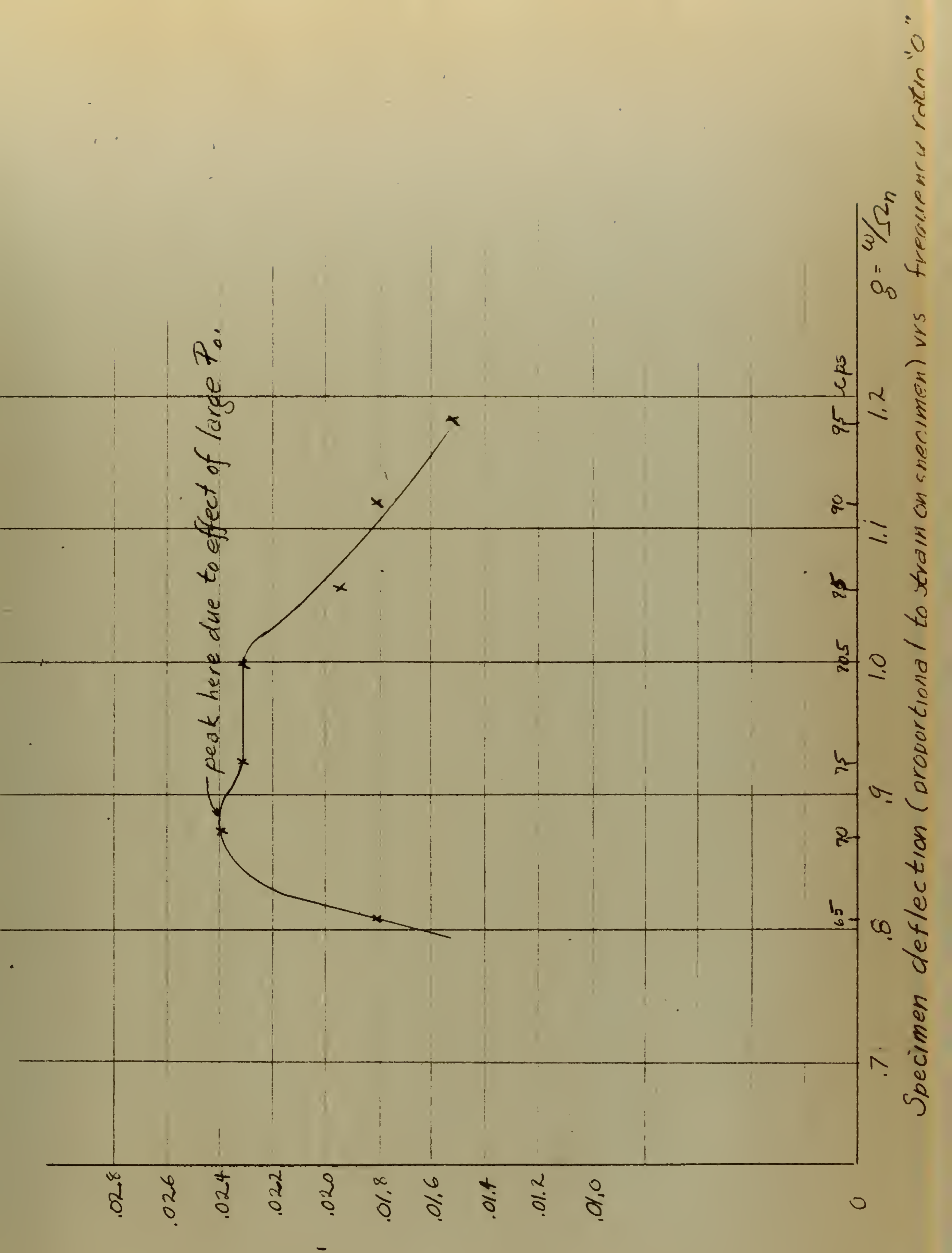
Comparison of calculated and measured values of $\frac{X_2 - X_1}{X_{st}}$.

Fig. 8



Watts
Watts
Watts
P_o
cps
$g = \omega/\Omega_n$
Driving Force P_o - lbs
150
125
100
75
50
25
60
50
40
30
20
10
.7
.8
.9
1.0
1.1
1.2
1.3
65
70
75
80.5
85
90
95

Fig 9



peak here due to effect of large P_o.
Specimen deflection (proportional to strain on specimen) vrs frequency ratio "g"
$g = \omega/\Omega_n$
.7
.8
.9
1.0
1.1
1.2
65
70
75
80.5
85
90
95
cps
.028
.026
.024
.022
.020
.018
.016
.014
.012
.010
0
$X_2 - X_1$

Fig 10

Thesis
P338
28468
Perkins
Method for determining fatigue characteristics of metals under combined bending and torsion.